# The informativity approach to data-driven analysis and control

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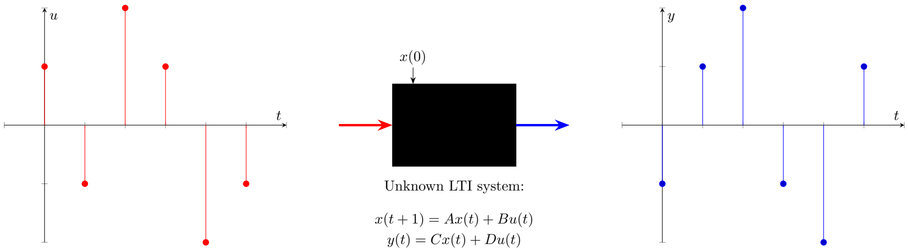
# Outline

a story of four parts

1. Background
2. Informativity framework
3. Example: stabilization using input-output data
4. Conclusions

# Background

# The fundamental lemma




Rearrange the measurements $(u(t), y(t))$ for $t = 0, 1, \dots, T-1$ into Hankel matrices:

$$\underbrace{\begin{bmatrix} \mathcal{H}_L(u) \\ \mathcal{H}_L(y) \end{bmatrix}}_{\mathcal{H}_L} = \begin{bmatrix} u(0) & u(1) & \cdots & u(T-L) \\ \vdots & \vdots & & \vdots \\ u(L-1) & u(L) & \cdots & u(T-1) \\ y(0) & y(1) & \cdots & y(T-L) \\ \vdots & \vdots & & \vdots \\ y(L-1) & y(L) & \cdots & y(T-1) \end{bmatrix}$$

Any vector

$$\begin{bmatrix} \bar{u}(0) \\ \vdots \\ \bar{u}(L-1) \\ \bar{y}(0) \\ \vdots \\ \bar{y}(L-1) \end{bmatrix} \in \operatorname{im} \mathcal{H}_L$$

is a trajectory.

# The fundamental lemma

Question: Under which conditions do the columns of $\mathcal{H}_L$ span ALL length-$L$ trajectories?

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## A note on persistency of excitation

Jan C. Willems[a], Paolo Rapisarda[b], Ivan Markovsky[a,*], Bart L.M. De Moor[a]

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**Abstract**

We prove that if a component of the response signal of a controllable linear time-invariant system is persistently exciting of sufficiently high order, then the windows of the signal span the full system behavior. This is then applied to obtain conditions

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The input $u$ is persistently exciting (PE) of order $k$ if $\mathcal{H}_k(u)$ has full row rank.

This requires: $T \geqslant (\dim(u) + 1)k - 1$.

The fundamental lemma: If $(A, B)$ is controllable and $u$ is PE of order $\dim(x) + L$ then $\operatorname{im} \mathcal{H}_L$ spans all length-$L$ trajectories the system can produce.

# The fundamental lemma

## some follow-up results

ON- AND OFF-LINE IDENTIFICATION OF LINEAR STATE SPACE MODELS

Marc Moonen*, Bart De Moor, Lieven Vandenberghe*, Joos Vandewalle
ESAT Katholieke Universiteit Leuven

Parametric model $(A, B, C, D)$ from $\mathcal{H}_L$

**Data-driven simulation and control**

Ivan Markovsky* and Paolo Rapisarda

*School of Electronics and Computer Science, University of Southampton, Southampton, UK*

(*Received 21 June 2007; final version received 24 January 2008*)

Classical linear time-invariant system simulation methods are based on a transfer function, impulse response, or input/state/output representation. We present a method for computing the response of a system to a given input and initial conditions directly from a trajectory of the system, without explicitly identifying the system from the

(Model-free) simulation of trajectories

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Data-Enabled Predictive Control: In the Shallows of the DeePC

Jeremy Coulson John Lygeros Florian Dörfler

***Abstract*—We consider the problem of optimal trajectory tracking for unknown systems. A novel data-enabled predictive control (DeePC) algorithm is presented that computes optimal and safe control policies using real-time feedback driving the** approaches usually require a large number of data samples to perform well, and are often sensitive to hyper-parameters leading to non-reproducible and highly variable

Prediction and optimization of trajectories

Formulas for Data-Driven Control: Stabilization, Optimality, and Robustness

Claudio De Persis and Pietro Tesi

***Abstract*—In a paper by Willems *et al.*, it was shown that persistently exciting data can be used to represent the input–output behavior of a linear system. Based on this** the achievements obtained in system identification. A major challenge is how to incorporate data-dependent stability and performance requirements in the control design procedure.

Robust data-driven state-feedback design

Julian Berberich[1], Anne Koch[1], Carsten W. Scherer[2], and Frank Allgöwer[1]

***Abstract*—We consider the problem of designing robust state-feedback controllers for discrete-time linear time-invariant systems, based directly on measured data. The proposed design** or an extension of [9] to certain classes of nonlinear systems [11]. Moreover, the recent work [12] derives a simple data-dependent closed-loop parametrization of LTI systems

Feedback controllers

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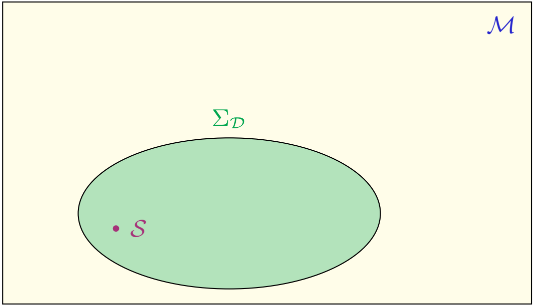
Some limitations of the fundamental lemma:

- For control, we may not need the entire (exact) restricted behavior. PE necessary?
- In general, it is not possible to find restricted behavior. Noisy data?

# The framework

# The point of view




$\mathcal{M}$: model class

$\mathcal{S}$: unknown system

$\mathcal{D}$: given data set

$\Sigma_{\mathcal{D}}$: set of consistent systems

$\mathcal{O}$: control objective

- Data-driven control := use the data set $\mathcal{D}$ to find a controller $\mathcal{C}$ that achieves $\mathcal{O}$ for the unknown system $\mathcal{S}$
- On the basis of $\mathcal{D}$ we cannot distinguish between systems in $\Sigma_{\mathcal{D}}$ so the only way to proceed is to find a controller that achieves $\mathcal{O}$ for all systems in $\Sigma_{\mathcal{D}}$
- Data $\mathcal{D}$ are informative for $\mathcal{O}$ $:\Longleftrightarrow$ $\exists$ controller $\mathcal{C}$ that achieves $\mathcal{O}$ for all systems in $\Sigma_{\mathcal{D}}$
- Type of robust control problem, where uncertainty stems from imperfect data

# Informativity approach

problems solved so far

| Problem | Data | Problem | Data |
|---|---|---|---|
| controllability | E-IS | reachability (conic constraints) | E-IO |
| observability | E-S | stability | N-S |
| stabilizability | E-IS | stabilizability | N-IS |
| stability | E-S | state feedback stabilization | N-IS |
| state feedback stabilization | E-IS | state feedback $\mathcal{H}_2$ control | N-IS |
| deadbeat controller | E-IS | dynamic feedback $\mathcal{H}_2$ control | N-IO |
| LQR | E-IS | state feedback $\mathcal{H}_\infty$ control | N-IS |
| suboptimal LQR | E-IS | dynamic feedback $\mathcal{H}_\infty$ control | N-IO |
| suboptimal $\mathcal{H}_2$ | E-IS | stability | N-IO |
| dynamic feedback stabilization | E-ISO | dynamic feedback stabilization | N-IO |
| dynamic feedback stabilization | E-IO | dissipativity | N-ISO |
| dissipativity | E-ISO | model reduction (balancing) | N-ISO |
| tracking and regulation | E-IS | structural properties | N-ISO |
| model reduction (moment matching) | E-IO | absolute stabilization Lur'e | N-ISO |

Table: Summary of results within the informativity approach, see also: "A Tutorial on the Informativity Framework for Data-Driven Control" (TuBT12).

## Stabilization using quadratic difference forms

# Input-output systems in AR form and data

## The model class and data

Model class of all of systems of known order $L$, input dimension $m$, output dimension $p$:

$$P(\sigma)\boldsymbol{y} = Q(\sigma)\boldsymbol{u} + \boldsymbol{v},$$

where $\boldsymbol{v}(t)$ is unknown additive noise, $\sigma$ is the shift operator $((\sigma \boldsymbol{f})(t) = \boldsymbol{f}(t+1))$ and $P, Q$ are polynomial matrices:

$$\begin{aligned} P(\xi) &= I\xi^L + P_{L-1}\xi^{L-1} + \cdots + P_1\xi + P_0, \\ Q(\xi) &= Q_{L-1}\xi^{L-1} + \cdots + Q_1\xi + Q_0, \end{aligned}$$

with unknown coefficients $P_0, P_1, \ldots, P_{L-1} \in \mathbb{R}^{p\times p}$, $Q_0, Q_1, \ldots, Q_{L-1} \in \mathbb{R}^{p\times m}$.

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We collect input-output data

$$u(0), u(1), \ldots, u(T),\ y(0), y(1), \ldots, y(T),$$

for $T \geqslant L$, generated by the (unknown) true system

$$P_s(\sigma)\boldsymbol{y} = Q_s(\sigma)\boldsymbol{u} + \boldsymbol{v}$$

within the model class.

# Input-output systems in AR form and data

## Assumption on the noise

The $\tau := T - L + 1$ noise samples $v(0), v(1), \ldots, v(T-L)$ are unknown. However, we assume that the real $p \times \tau$ matrix

$$V := \begin{bmatrix} v(0) & v(1) & \cdots & v(T-L) \end{bmatrix},$$

satisfies the quadratic matrix inequality (QMI)

$$\begin{bmatrix} I \\ V^\top \end{bmatrix}^\top \Pi \begin{bmatrix} I \\ V^\top \end{bmatrix} \geqslant 0. \tag{1}$$

Here $\Pi \in \mathbb{S}^{p+\tau}$ is a given (known) partitioned matrix

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{21} & \Pi_{22} \end{bmatrix},$$

with $\Pi_{22} < 0$. Notation: $V^\top \in \mathcal{Z}_\tau(\Pi)$.

# Input-output systems in AR form and data

## All systems consistent with the data

Notation: $R(\xi) = \begin{bmatrix} -Q(\xi) & P(\xi) \end{bmatrix}$, $\boldsymbol{w} = \begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{y} \end{bmatrix}$, $q = m + p$,

Then $P(\sigma)\boldsymbol{y} = Q(\sigma)\boldsymbol{u} + \boldsymbol{v}$ can be written as $R(\sigma)\boldsymbol{w} = \boldsymbol{v}$.

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(Unknown) coefficient matrix of $R(\xi)$: the $p \times qL$ matrix

$$\tilde{R} := \begin{bmatrix} -Q_0 & P_0 & -Q_1 & P_1 & \cdots & -Q_{L-1} & P_{L-1} \end{bmatrix}$$

Arrange the data $u(0), u(1), \ldots, u(T), y(0), y(1), \ldots, y(T)$ into the vectors

$$w(t) = \begin{bmatrix} u(t) \\ y(t) \end{bmatrix}.$$

(Adapted) Hankel matrix asociated with the data:

$$H(w) := \left[\begin{array}{cccc} w(0) & w(1) & \cdots & w(T-L) \\ \vdots & \vdots & & \vdots \\ w(L-1) & w(L) & \cdots & w(T-1) \\ \hdashline y(L) & y(L+1) & \cdots & y(T) \end{array}\right] = \left[\begin{array}{c} H_1(w) \\ \hdashline H_2(w) \end{array}\right]$$

# Input-output systems in AR form and data

## All systems compatible with the data

Observation: if a matrix $\tilde{R}$ satisfies the linear equation

$$\begin{bmatrix} \tilde{R} & I \end{bmatrix} \begin{bmatrix} H_1(w) \\ H_2(w) \end{bmatrix} = V \qquad (*)$$

for some $V^\top \in \mathcal{Z}_\tau(\Pi)$ then the data $w(0), w(1), \ldots, w(T)$ could have been generated by the system $R(\sigma)\boldsymbol{w} = \boldsymbol{v}$.

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Definition: If $\tilde{R}$ satisfies $(*)$ for some $V^\top \in \mathcal{Z}_\tau(\Pi)$, we call the AR system with coefficient matrix $\tilde{R}$ consistent with the data.

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Fact: The system $R(\sigma)\boldsymbol{w} = \boldsymbol{v}$ is consistent with the data if and only if

$$\begin{bmatrix} I \\ \tilde{R}^\top \end{bmatrix}^\top \underbrace{\begin{bmatrix} I & H_2(w) \\ 0 & H_1(w) \end{bmatrix} \Pi \begin{bmatrix} I & H_2(w) \\ 0 & H_1(w) \end{bmatrix}^\top}_{=:N} \begin{bmatrix} I \\ \tilde{R}^\top \end{bmatrix} \geqslant 0.$$

Stability and quadratic difference forms

# Stability of autonomous systems

Some facts on stability and Lyapunov theory for autonomous systems represented by AR models:

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Definition: Let $P(\xi)$ be a nonsingular polynomial matrix. The system $P(\sigma)\boldsymbol{y} = 0$ is (asymptotically) stable if $\boldsymbol{y}(t) \to 0$ as $t \to \infty$ for all solutions $\boldsymbol{y}$ on $\mathbb{Z}_+$.

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Behavior: $\mathcal{B}(P) := \{\boldsymbol{y} : \mathbb{Z}_+ \to \mathbb{R}^p \mid P(\sigma)\boldsymbol{y} = 0\}$

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Stability of autonomous AR systems can be characterized in terms of quadratic difference forms (QDFs) along the behavior[1,2].

We will discuss QDFs now...

[1] Willems and Trentelman, "On quadratic differential forms" (1998).

[2] Kojima and Takaba, "A generalized Lyapunov stability theorem for discrete-time systems based on quadratic difference forms" (2005).

# Quadratic difference forms

A quadratic difference form (QDF): operator $Q_\Phi$ mapping $\boldsymbol{w} : \mathbb{Z}_+ \to \mathbb{R}^q$ to $Q_\Phi(\boldsymbol{w}) : \mathbb{Z}_+ \to \mathbb{R}$, defined by

$$Q_\Phi(\boldsymbol{w})(t) := \sum_{k,\ell=0}^{N} \boldsymbol{w}(t+k)^\top \Phi_{k,\ell}\, \boldsymbol{w}(t+\ell).$$

Here, $N$ and $q$ are positive integers and $\Phi_{i,j} \in \mathbb{R}^{q\times q}$. We assume

$$\Phi := \begin{bmatrix} \Phi_{0,0} & \Phi_{0,1} & \cdots & \Phi_{0,N} \\ \Phi_{1,0} & \Phi_{1,1} & \cdots & \Phi_{1,N} \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_{N,0} & \Phi_{N,1} & \cdots & \Phi_{N,N} \end{bmatrix} \in \mathbb{S}^{q(N+1)}.$$

$\Phi$ is called a coefficient matrix of the QDF.
The degree $\deg(Q_\Phi)$ of the QDF: the smallest integer $d$ such that $\Phi_{ij} = 0$ for all $i > d$ or $j > d$.
The coefficient matrix is not unique. If $\deg(Q_\Phi) = d$ it allows a coefficient matrix in $\mathbb{S}^{q(d+1)}$.

# Quadratic difference forms

For a given QDF $Q_\Phi$, its rate of change along $\boldsymbol{w} : \mathbb{Z}_+ \to \mathbb{R}^q$ is

$$Q_\Phi(\boldsymbol{w})(t+1) - Q_\Phi(\boldsymbol{w})(t).$$

This defines a QDF itself. Indeed, $Q_{\nabla\Phi}(\boldsymbol{w})(t) = Q_\Phi(\boldsymbol{w})(t+1) - Q_\Phi(\boldsymbol{w})(t)$ where

$$\nabla\Phi := \begin{bmatrix} 0_{q\times q} & 0 \\ 0 & \Phi \end{bmatrix} - \begin{bmatrix} \Phi & 0 \\ 0 & 0_{q\times q} \end{bmatrix} \in \mathbb{S}^{q(N+2)}.$$

---

$R(\xi)$ real $p \times q$ polynomial matrix. AR system represented by $R(\sigma)\boldsymbol{w} = 0$.
The behavior of this system is denoted by $\mathcal{B}(R)$.

- $Q_\Phi$ is called nonnegative if $Q_\Phi(\boldsymbol{w}) \geqslant 0$ for all $\boldsymbol{w} : \mathbb{Z}_+ \to \mathbb{R}^q$. We denote this as $Q_\Phi \geqslant 0$. Equivalent with $\Phi \geqslant 0$.
- $Q_\Phi$ is called nonnegative on $\mathcal{B}(R)$ if $Q_\Phi(\boldsymbol{w}) \geqslant 0$ for all $\boldsymbol{w} \in \mathcal{B}(R)$. Notation: $Q_\Phi \geqslant 0$ on $\mathcal{B}(R)$.
- $Q_\Phi$ is called positive if $Q_\Phi \geqslant 0$ and, in addition, $Q_\Phi(\boldsymbol{w}) = 0$ if and only if $\boldsymbol{w} = 0$. This is denoted as $Q_\Phi > 0$.
- It is called positive on $\mathcal{B}(R)$ if, in addition, $Q_\Phi(\boldsymbol{w}) = 0$ if and only if $\boldsymbol{w} = 0$. Notation: $Q_\Phi > 0$ on $\mathcal{B}(R)$. Likewise: nonpositivity and negativity on $\mathcal{B}(R)$.

## Stability of autonomous systems

Consider the autonomous AR system represented by $P(\sigma)\boldsymbol{y} = 0$, where we recall that $P(\xi) = I\xi^L + P_{L-1}\xi^{L-1} + \cdots + P_1\xi + P_0$.

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Lemma: The autonomous system $P(\sigma)\boldsymbol{y} = 0$ of order $L$ is stable if and only if there exists a QDF $Q_\Psi$ of degree at most $L-1$ such that $Q_\Psi \geqslant 0$ and $Q_{\nabla\Psi} < 0$ on $\mathcal{B}(P)$.

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The QDF $Q_\Psi$ is called a Lyapunov function.

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In terms of the coefficient matrix $\tilde{P} = \begin{bmatrix} P_0 & P_1 & \cdots & P_{L-1} \end{bmatrix} \in \mathbb{R}^{p\times pL}$ this can be translated to:

Theorem: The autonomous system $P(\sigma)\boldsymbol{y} = 0$ of order $L$ is stable if and only if there exists $\Psi \in \mathbb{S}^{pL}$, $\Psi \geqslant 0$, such that

$$\begin{bmatrix} I \\ -\tilde{P} \end{bmatrix}^\top \left( \begin{bmatrix} 0_p & 0 \\ 0 & \Psi \end{bmatrix} - \begin{bmatrix} \Psi & 0 \\ 0 & 0_p \end{bmatrix} \right) \begin{bmatrix} I \\ -\tilde{P} \end{bmatrix} < 0. \qquad \text{(QMI with } \tilde{P} \text{ and } \Psi\text{)}$$

Any such $\Psi$ defines a Lyapunov function $Q_\Psi$.

# Data-driven controller synthesis

# Data-driven stabilization

## Dynamic output feedback controllers

Feedback controller for $P(\sigma)\boldsymbol{y} = Q(\sigma)\boldsymbol{u} + \boldsymbol{v}$ of the form

$$G(\sigma)\boldsymbol{u} = F(\sigma)\boldsymbol{y}$$

with

$$\begin{aligned} G(\xi) &= I\xi^L + G_{L-1}\xi^{L-1} + \cdots + G_1\xi + G_0, \\ F(\xi) &= F_{L-1}\xi^{L-1} + \cdots + F_1\xi + F_0. \end{aligned}$$

Note: the leading coefficient matrix of $G(\xi)$ is the $m \times m$ identity matrix and $G_i \in \mathbb{R}^{m\times m}$, $F_i \in \mathbb{R}^{m\times p}$ for $i = 0, 1, \ldots, L-1$.
Closed loop system given by

$$\begin{bmatrix} G(\sigma) & -F(\sigma) \\ -Q(\sigma) & P(\sigma) \end{bmatrix} \begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{y} \end{bmatrix} = \begin{bmatrix} 0 \\ I \end{bmatrix} \boldsymbol{v}.$$

The leading coefficient matrix is the $q \times q$ identity matrix. We call the controller a stabilizing controller if the closed loop system is stable in the sense that if $\boldsymbol{v} = 0$ then $\boldsymbol{u}(t)$ and $\boldsymbol{y}(t)$ tend to $0$ as $t$ tends to infinity.

# Data-driven stabilization

Denote $C(\xi) := \begin{bmatrix} G(\xi) & -F(\xi) \end{bmatrix}$ and
$\tilde{C} := \begin{bmatrix} G_0 & -F_0 & G_1 & -F_1 & \cdots & G_{L-1} & -F_{L-1} \end{bmatrix} \in \mathbb{R}^{m \times qL}$.

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Lemma: The closed loop system is stable if and only if there exists $\Psi \in \mathbb{S}^{qL}$ such that $\Psi \geqslant 0$ and

$$\begin{bmatrix} I \\ -\tilde{C} \\ -\tilde{R} \end{bmatrix}^\top \left( \begin{bmatrix} 0_q & 0 \\ 0 & \Psi \end{bmatrix} - \begin{bmatrix} \Psi & 0 \\ 0 & 0_q \end{bmatrix} \right) \begin{bmatrix} I \\ -\tilde{C} \\ -\tilde{R} \end{bmatrix} < 0. \qquad \text{(closed loop stability)}$$

Moreover, in that case, $\Psi > 0$.

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Definition: The data $u(0), u(1), \ldots, u(T), y(0), y(1), \ldots, y(T)$ are called informative for quadratic stabilization if there exist $\tilde{C} \in \mathbb{R}^{m \times qL}$ and $\Psi \in \mathbb{S}^{qL}$ such that $\Psi \geqslant 0$ and QMI (closed loop stability) holds for all $\tilde{R}$ that satisfy

$$\begin{bmatrix} I \\ \tilde{R}^\top \end{bmatrix}^\top N \begin{bmatrix} I \\ \tilde{R}^\top \end{bmatrix} \geqslant 0.$$

In other words: there exists a controller $G(\sigma)\boldsymbol{u} = F(\sigma)\boldsymbol{y}$ and $\Psi \in \mathbb{S}^{qL}$ such that $Q_\Psi$ is a common Lyapunov function for all closed loop systems obtained by interconnecting the controller with a system compatible with the data.

# Data-driven stabilization

## The main tools

Informativity for quadratic stabilization thus means that there exists a controller $\tilde{C}$ and (common) Lyapunov function $\Psi$ such that all solutions to the QMI

$$\begin{bmatrix} I \\ \tilde{R}^\top \end{bmatrix}^\top N \begin{bmatrix} I \\ \tilde{R}^\top \end{bmatrix} \geqslant 0$$

also satisfy the QMI

$$\begin{bmatrix} I \\ -\tilde{C} \\ -\tilde{R} \end{bmatrix}^\top \left( \begin{bmatrix} 0_q & 0 \\ 0 & \Psi \end{bmatrix} - \begin{bmatrix} \Psi & 0 \\ 0 & 0_q \end{bmatrix} \right) \begin{bmatrix} I \\ -\tilde{C} \\ -\tilde{R} \end{bmatrix} < 0.$$

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To characterize informativity we use two main ingredients:

- A dualization and projection step
- A matrix version of Yakubovich's S-lemma[3]

[3] van Waarde *et al.*, "Quadratic matrix inequalities with applications to data-based control", *https://arxiv.org/abs/2203.12959*, 2022.

# Data-driven stabilization

Let $J := \begin{bmatrix} 0_{q(L-1)\times q} & I_{q(L-1)} \end{bmatrix}$ and define the $2qL \times 2qL$ matrix $\bar{N}$ by

$$\bar{N} := \begin{bmatrix} \begin{bmatrix} 0 & 0 & -I_p \end{bmatrix} & 0 \\ 0 & I_{qL} \end{bmatrix}^\top N \begin{bmatrix} \begin{bmatrix} 0 & 0 & -I_p \end{bmatrix} & 0 \\ 0 & I_{qL} \end{bmatrix}.$$

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Theorem[4]: Assume that $H_1(w)$ has full row rank. The data $u(0), u(1), \ldots, u(T)$, $y(0), y(1), \ldots, y(T)$ are informative for quadratic stabilization if and only if there exist matrices $\tilde{D} \in \mathbb{R}^{m\times qL}$ and $\Phi \in \mathbb{S}^{qL}$ such that

$$\begin{bmatrix} \Phi & \begin{bmatrix} J\Phi \\ \tilde{D} \\ 0 \end{bmatrix} & \begin{bmatrix} J\Phi \\ \tilde{D} \\ 0 \end{bmatrix} \\ \begin{bmatrix} J\Phi \\ \tilde{D} \\ 0 \end{bmatrix}^\top & -\Phi & 0 \\ \begin{bmatrix} J\Phi \\ \tilde{D} \\ 0 \end{bmatrix}^\top & 0 & \Phi \end{bmatrix} - \begin{bmatrix} \bar{N} & 0 \\ 0 & 0_{qL} \end{bmatrix} > 0.$$

In that case, the controller with coefficient matrix $\tilde{C} := -\tilde{D}\Phi^{-1}$ stabilizes all systems that are compatible with the data. Moreover, the QDF $Q_\Psi$ with $\Psi := \Phi^{-1}$ is a common Lyapunov function for all closed loop systems.

[4] van Waarde *et al.*, "A behavioral approach to data-driven control with noisy input-output data", arxiv.org/abs/2206.08408, 2022.

# Conclusions

# Conclusions

## summary and future work

General framework of data informativity:

1. Set of systems explaining the data
2. Control all systems in this set

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Conditions for data informativity for stabilization, LQR, tracking and regulation,...

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Control design via linear matrix inequalities

- Controllers are obtained directly using a finite batch of (noisy) data
- Approach relies on behavioral theory (QDFs) and robust control (matrix S-lemma)
- Nonconservative design

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Ongoing and future work:

- (Common) Lyapunov functions (also see WeBT05.5)
- Online experiment design (PhD work of Amir Shakouri)

Thank you!