Data-driven stabilization using prior knowledge on controllability and stabilizability

O'Higgins Seminar on Optimization
4 December 2025

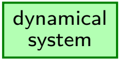
Henk van Waarde

Systems, Control and Optimization Group

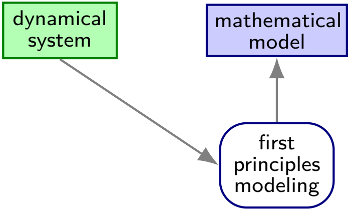
Bernoulli Institute for Mathematics, Computer Science and Artificial Intelligence

University of Groningen

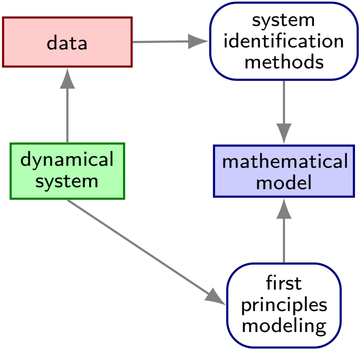
What is data-driven control?

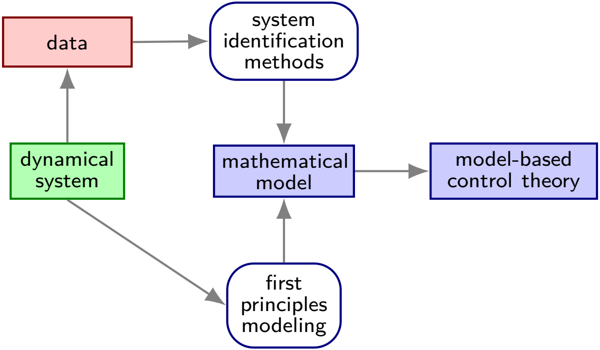


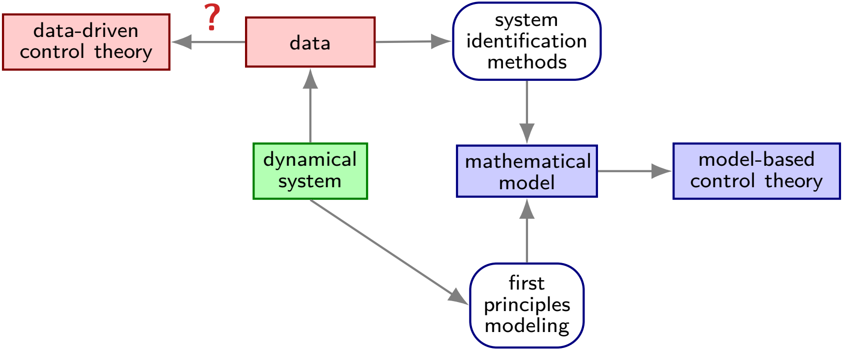
What is data-driven control?



What is data-driven control?




data
system identification methods
dynamical system
mathematical model
model-based control theory
first principles modeling


data-driven
control theory
?
data
system
identification
methods
dynamical
system
mathematical
model
model-based
control theory
first
principles
modeling

1. Data-driven control: Finding a controller directly from data.

1. Data-driven control: Finding a controller directly from data.
   - Less computations due to bypassing system identification.

1. Data-driven control: Finding a controller directly from data.
   - Less computations due to bypassing system identification.
   - Finding a feedback law when identification is not feasible.

1. Data-driven control: Finding a controller directly from data.
   - Less computations due to bypassing system identification.
   - Finding a feedback law when identification is not feasible.
2. Data-driven stabilization: Finding a stabilizing controller directly from data.

1. Data-driven control: Finding a controller directly from data.
   - Less computations due to bypassing system identification.
   - Finding a feedback law when identification is not feasible.
2. Data-driven stabilization: Finding a stabilizing controller directly from data.
   - Using data generated by persistently exciting input (De Persis and Tesi, 2020)[1].

[1] C. De Persis, P. Tesi, "Formulas for data-driven control: Stabilization, optimality, and robustness," *IEEE Transactions on Automatic Control*, vol. 65, no. 3, pp. 909–924, 2020.

1. Data-driven control: Finding a controller directly from data.
   - Less computations due to bypassing system identification.
   - Finding a feedback law when identification is not feasible.
2. Data-driven stabilization: Finding a stabilizing controller directly from data.
   - Using data generated by persistently exciting input (De Persis and Tesi, 2020).
   - Necessary and sufficient conditions on the data (van Waarde et al., 2020)[2].

[2] H. J. van Waarde, J. Eising, H. L. Trentelman, M. K. Camlibel, "Data informativity: A new perspective on data-driven analysis and control," *IEEE Transactions on Automatic Control*, vol. 65, no. 11, pp. 4753–4768, 2020.

1. Data-driven control: Finding a controller directly from data.
   - Less computations due to bypassing system identification.
   - Finding a feedback law when identification is not feasible.
2. Data-driven stabilization: Finding a stabilizing controller directly from data.
   - Using data generated by persistently exciting input (De Persis and Tesi, 2020).
   - Necessary and sufficient conditions on the data (van Waarde et al., 2020).
3. Prior knowledge in data-driven control:

1. Data-driven control: Finding a controller directly from data.
   - Less computations due to bypassing system identification.
   - Finding a feedback law when identification is not feasible.
2. Data-driven stabilization: Finding a stabilizing controller directly from data.
   - Using data generated by persistently exciting input (De Persis and Tesi, 2020).
   - Necessary and sufficient conditions on the data (van Waarde et al., 2020).
3. Prior knowledge in data-driven control:
   - Known bounds on the system parameters (Berberich et al., 2020)[3].

[3] J. Berberich, C. W. Scherer, F. Allgöwer, "Combining prior knowledge and data for robust controller design," *IEEE Transactions on Automatic Control*, vol. 68, no. 8, pp. 4618–4633, 2022.

1. Data-driven control: Finding a controller directly from data.
   - Less computations due to bypassing system identification.
   - Finding a feedback law when identification is not feasible.
2. Data-driven stabilization: Finding a stabilizing controller directly from data.
   - Using data generated by persistently exciting input (De Persis and Tesi, 2020).
   - Necessary and sufficient conditions on the data (van Waarde et al., 2020).
3. Prior knowledge in data-driven control:
   - Known bounds on the system parameters (Berberich et al., 2020).
   - Some of the parameters are exactly known (Huang et al., 2025)[4].

[4] H. Huang, M. K. Camlibel, R. Carloni, H. J. van Waarde, "Data-driven stabilization of polynomial systems using density functions," *arXiv preprint*, 2025.

1. Data-driven control: Finding a controller directly from data.
   - Less computations due to bypassing system identification.
   - Finding a feedback law when identification is not feasible.
2. Data-driven stabilization: Finding a stabilizing controller directly from data.
   - Using data generated by persistently exciting input (De Persis and Tesi, 2020).
   - Necessary and sufficient conditions on the data (van Waarde et al., 2020).
3. Prior knowledge in data-driven control:
   - Known bounds on the system parameters (Berberich et al., 2020).
   - Some of the parameters are exactly known (Huang et al., 2025).
4. Prior knowledge in system identification:

1. Data-driven control: Finding a controller directly from data.
   - Less computations due to bypassing system identification.
   - Finding a feedback law when identification is not feasible.
2. Data-driven stabilization: Finding a stabilizing controller directly from data.
   - Using data generated by persistently exciting input (De Persis and Tesi, 2020).
   - Necessary and sufficient conditions on the data (van Waarde et al., 2020).
3. Prior knowledge in data-driven control:
   - Known bounds on the system parameters (Berberich et al., 2020).
   - Some of the parameters are exactly known (Huang et al., 2025).
4. Prior knowledge in system identification:
   - Stability (van Gestel et al., 2002, Lacy & Bernstein, 2003)
   - Eigenvalue constraints (Miller & De Callafon, 2013)
   - Positivity (De Santis & Farina, 2002)
   - Passivity (Goethals et al., 2003, Shali & van Waarde, 2024)

1. Data-driven control: Finding a controller directly from data.
   - Less computations due to bypassing system identification.
   - Finding a feedback law when identification is not feasible.
2. Data-driven stabilization: Finding a stabilizing controller directly from data.
   - Using data generated by persistently exciting input (De Persis and Tesi, 2020).
   - Necessary and sufficient conditions on the data (van Waarde et al., 2020).
3. Prior knowledge in data-driven control:
   - Known bounds on the system parameters (Berberich et al., 2020).
   - Some of the parameters are exactly known (Huang et al., 2025).
4. Prior knowledge in system identification:
   - Stability (van Gestel et al., 2002, Lacy & Bernstein, 2003)
   - Eigenvalue constraints (Miller & De Callafon, 2013)
   - Positivity (De Santis & Farina, 2002)
   - Passivity (Goethals et al., 2003, Shali & van Waarde, 2024)

This talk: data-driven stabilization using prior knowledge on

- controllability
- stabilizability

Background on data-driven stabilization

Problem formulation

Controllability as prior knowledge

Stabilizability as prior knowledge

Experiment design

Conclusions

Consider the discrete-time system

$$x(t+1) = A_{\text{true}}x(t) + B_{\text{true}}u(t),$$

referred to as the true system, where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$.

Consider the discrete-time system

$$x(t+1) = A_{\text{true}}x(t) + B_{\text{true}}u(t),$$

referred to as the true system, where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$.

The system matrices $(A_{\text{true}}, B_{\text{true}}) \in \mathcal{M} := \mathbb{R}^{n\times n} \times \mathbb{R}^{n\times m}$ are unknown.

Consider the discrete-time system

$$x(t+1) = A_{\mathrm{true}} x(t) + B_{\mathrm{true}} u(t),$$

referred to as the true system, where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$.

The system matrices $(A_{\mathrm{true}}, B_{\mathrm{true}}) \in \mathcal{M} := \mathbb{R}^{n \times n} \times \mathbb{R}^{n \times m}$ are unknown.

We have access to input-state data of the form

$$\mathcal{D} := \left(\begin{bmatrix} u(0) & u(1) & \cdots & u(T-1) \end{bmatrix}, \begin{bmatrix} x(0) & x(1) & \cdots & x(T) \end{bmatrix} \right).$$

Consider the discrete-time system

$$x(t+1) = A_{\text{true}}x(t) + B_{\text{true}}u(t),$$

referred to as the true system, where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$.

The system matrices $(A_{\text{true}}, B_{\text{true}}) \in \mathcal{M} := \mathbb{R}^{n\times n} \times \mathbb{R}^{n\times m}$ are unknown.

We have access to input-state data of the form

$$\mathcal{D} := \left(\begin{bmatrix} u(0) & u(1) & \cdots & u(T-1)\end{bmatrix}, \begin{bmatrix} x(0) & x(1) & \cdots & x(T)\end{bmatrix}\right).$$

Overall goal: Given the data $\mathcal{D}$, find a state feedback controller $u(t) = Kx(t)$ such that

$$x(t+1) = (A_{\text{true}} + B_{\text{true}}K)x(t)$$

is asymptotically stable.

Consider the discrete-time system

$$x(t+1) = A_{\text{true}}x(t) + B_{\text{true}}u(t),$$

referred to as the true system, where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$.

The system matrices $(A_{\text{true}}, B_{\text{true}}) \in \mathcal{M} := \mathbb{R}^{n\times n} \times \mathbb{R}^{n\times m}$ are unknown.

We have access to input-state data of the form

$$\mathcal{D} := \left(\begin{bmatrix} u(0) & u(1) & \cdots & u(T-1) \end{bmatrix}, \begin{bmatrix} x(0) & x(1) & \cdots & x(T) \end{bmatrix}\right).$$

Overall goal: Given the data $\mathcal{D}$, find a state feedback controller $u(t) = Kx(t)$ such that

$$x(t+1) = (A_{\text{true}} + B_{\text{true}}K)x(t)$$

is asymptotically stable.

Equivalently: Given $\mathcal{D}$, find $K \in \mathbb{R}^{m\times n}$ such that $A_{\text{true}} + B_{\text{true}}K$ is Schur.

We define the data matrices

$$\begin{aligned} X_- &:= \begin{bmatrix} x(0) & x(1) & \cdots & x(T-1) \end{bmatrix} \\ U_- &:= \begin{bmatrix} u(0) & u(1) & \cdots & u(T-1) \end{bmatrix} \\ X_+ &:= \begin{bmatrix} x(1) & x(2) & \cdots & x(T) \end{bmatrix}. \end{aligned}$$

We define the data matrices

$$X_- := \begin{bmatrix} x(0) & x(1) & \cdots & x(T-1) \end{bmatrix}$$
$$U_- := \begin{bmatrix} u(0) & u(1) & \cdots & u(T-1) \end{bmatrix}$$
$$X_+ := \begin{bmatrix} x(1) & x(2) & \cdots & x(T) \end{bmatrix}.$$

The true system satisfies

$$X_+ = A_{\text{true}} X_- + B_{\text{true}} U_-.$$

We define the data matrices

$$\begin{aligned} X_- &:= \begin{bmatrix} x(0) & x(1) & \cdots & x(T-1) \end{bmatrix} \\ U_- &:= \begin{bmatrix} u(0) & u(1) & \cdots & u(T-1) \end{bmatrix} \\ X_+ &:= \begin{bmatrix} x(1) & x(2) & \cdots & x(T) \end{bmatrix}. \end{aligned}$$

The true system satisfies

$$X_+ = A_{\text{true}} X_- + B_{\text{true}} U_-.$$

We define the set of all data-consistent systems as

$$\Sigma_{\mathcal{D}} := \{(A, B) \in \mathcal{M} \mid X_+ = AX_- + BU_-\}.$$

We define the data matrices

$$\begin{aligned} X_- &:= \begin{bmatrix} x(0) & x(1) & \cdots & x(T-1) \end{bmatrix} \\ U_- &:= \begin{bmatrix} u(0) & u(1) & \cdots & u(T-1) \end{bmatrix} \\ X_+ &:= \begin{bmatrix} x(1) & x(2) & \cdots & x(T) \end{bmatrix}. \end{aligned}$$

The true system satisfies

$$X_+ = A_{\text{true}} X_- + B_{\text{true}} U_-.$$

We define the set of all data-consistent systems as

$$\Sigma_{\mathcal{D}} := \{(A, B) \in \mathcal{M} \mid X_+ = AX_- + BU_-\}.$$

Data-driven stabilization problem: Given the data $\mathcal{D}$, find $K \in \mathbb{R}^{m \times n}$ such that $A + BK$ is Schur for all $(A, B) \in \Sigma_{\mathcal{D}}$.

Definition: We say the data $\mathcal{D}$ are informative for stabilization if there exists a $K \in \mathbb{R}^{m \times n}$ such that $A + BK$ is Schur for all $(A, B) \in \Sigma_{\mathcal{D}}$.

Definition: We say the data $\mathcal{D}$ are informative for stabilization if there exists a $K \in \mathbb{R}^{m\times n}$ such that $A + BK$ is Schur for all $(A, B) \in \Sigma_{\mathcal{D}}$.

Proposition (vW et al., 2020): $\mathcal{D}$ are informative for stabilization if and only if $\operatorname{rank} X_- = n$ and there exists a right inverse $X_-^\dagger$ such that $X_+ X_-^\dagger$ is Schur.

Definition: We say the data $\mathcal{D}$ are informative for stabilization if there exists a $K \in \mathbb{R}^{m\times n}$ such that $A+BK$ is Schur for all $(A,B) \in \Sigma_{\mathcal{D}}$.

Proposition (vW et al., 2020): $\mathcal{D}$ are informative for stabilization if and only if $\operatorname{rank} X_- = n$ and there exists a right inverse $X_-^\dagger$ such that $X_+X_-^\dagger$ is Schur.
In this case, $K := U_-X_-^\dagger$ is a stabilizing feedback gain for all systems in $\Sigma_{\mathcal{D}}$.

Definition: We say the data $\mathcal{D}$ are informative for stabilization if there exists a $K \in \mathbb{R}^{m\times n}$ such that $A + BK$ is Schur for all $(A, B) \in \Sigma_{\mathcal{D}}$.

Proposition (vW et al., 2020): $\mathcal{D}$ are informative for stabilization if and only if $\operatorname{rank} X_- = n$ and there exists a right inverse $X_-^\dagger$ such that $X_+X_-^\dagger$ is Schur. In this case, $K := U_-X_-^\dagger$ is a stabilizing feedback gain for all systems in $\Sigma_{\mathcal{D}}$.

Example: Consider the data $X = \begin{bmatrix} 1 & 0.5 & -0.25 \\ 0 & 1 & 1 \end{bmatrix}$ and $U_- = \begin{bmatrix} -1 & -1 \end{bmatrix}$.

Definition: We say the data $\mathcal{D}$ are informative for stabilization if there exists a $K \in \mathbb{R}^{m \times n}$ such that $A + BK$ is Schur for all $(A, B) \in \Sigma_{\mathcal{D}}$.

Proposition (vW et al., 2020): $\mathcal{D}$ are informative for stabilization if and only if $\operatorname{rank} X_- = n$ and there exists a right inverse $X_-^\dagger$ such that $X_+ X_-^\dagger$ is Schur. In this case, $K := U_- X_-^\dagger$ is a stabilizing feedback gain for all systems in $\Sigma_{\mathcal{D}}$.

Example: Consider the data $X = \begin{bmatrix} 1 & 0.5 & -0.25 \\ 0 & 1 & 1 \end{bmatrix}$ and $U_- = \begin{bmatrix} -1 & -1 \end{bmatrix}$.

Thus, $X_- = \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}$ and $X_+ = \begin{bmatrix} 0.5 & -0.25 \\ 1 & 1 \end{bmatrix}$.

Definition: We say the data $\mathcal{D}$ are informative for stabilization if there exists a $K \in \mathbb{R}^{m \times n}$ such that $A + BK$ is Schur for all $(A, B) \in \Sigma_{\mathcal{D}}$.

Proposition (vW et al., 2020): $\mathcal{D}$ are informative for stabilization if and only if $\operatorname{rank} X_- = n$ and there exists a right inverse $X_-^\dagger$ such that $X_+ X_-^\dagger$ is Schur.
In this case, $K := U_- X_-^\dagger$ is a stabilizing feedback gain for all systems in $\Sigma_{\mathcal{D}}$.

Example: Consider the data $X = \begin{bmatrix} 1 & 0.5 & -0.25 \\ 0 & 1 & 1 \end{bmatrix}$ and $U_- = \begin{bmatrix} -1 & -1 \end{bmatrix}$.

Thus, $X_- = \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}$ and $X_+ = \begin{bmatrix} 0.5 & -0.25 \\ 1 & 1 \end{bmatrix}$.

The matrix $X_+ X_-^\dagger = X_+ X_-^{-1} = \begin{bmatrix} 0.5 & -0.5 \\ 1 & 0.5 \end{bmatrix}$ is Schur (eigenvalues $\frac{1}{2}(1 \pm \sqrt{2} i)$).

Definition: We say the data $\mathcal{D}$ are informative for stabilization if there exists a $K \in \mathbb{R}^{m \times n}$ such that $A + BK$ is Schur for all $(A, B) \in \Sigma_{\mathcal{D}}$.

Proposition (vW et al., 2020): $\mathcal{D}$ are informative for stabilization if and only if $\operatorname{rank} X_- = n$ and there exists a right inverse $X_-^\dagger$ such that $X_+ X_-^\dagger$ is Schur.
In this case, $K := U_- X_-^\dagger$ is a stabilizing feedback gain for all systems in $\Sigma_{\mathcal{D}}$.

Example: Consider the data $X = \begin{bmatrix} 1 & 0.5 & -0.25 \\ 0 & 1 & 1 \end{bmatrix}$ and $U_- = \begin{bmatrix} -1 & -1 \end{bmatrix}$.

Thus, $X_- = \begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}$ and $X_+ = \begin{bmatrix} 0.5 & -0.25 \\ 1 & 1 \end{bmatrix}$.

The matrix $X_+ X_-^\dagger = X_+ X_-^{-1} = \begin{bmatrix} 0.5 & -0.5 \\ 1 & 0.5 \end{bmatrix}$ is Schur (eigenvalues $\frac{1}{2}(1 \pm \sqrt{2}i)$).

The controller $K := U_- X_-^\dagger = \begin{bmatrix} -1 & -\frac{1}{2} \end{bmatrix}$ stabilizes all systems in

$$\Sigma_{\mathcal{D}} = \left\{ \left(\begin{bmatrix} 1.5 + a & 0.5a \\ 1 + b & 0.5 + 0.5b \end{bmatrix}, \begin{bmatrix} 1 + a \\ b \end{bmatrix} \right) \mid a, b \in \mathbb{R} \right\}.$$

Definition: We say the data $\mathcal{D}$ are informative for stabilization if there exists a $K \in \mathbb{R}^{m\times n}$ such that $A+BK$ is Schur for all $(A,B)\in\Sigma_{\mathcal{D}}$.

Proposition (vW et al., 2020): $\mathcal{D}$ are informative for stabilization if and only if $\operatorname{rank} X_- = n$ and there exists a right inverse $X_-^\dagger$ such that $X_+X_-^\dagger$ is Schur.
In this case, $K := U_-X_-^\dagger$ is a stabilizing feedback gain for all systems in $\Sigma_{\mathcal{D}}$.

Remark: If the data are informative, a suitable $X_-^\dagger$ may be computed as follows:

- Find $\Theta \in \mathbb{R}^{T\times n}$ solving the linear matrix inequality (LMI):

$$X_-\Theta = \Theta^\top X_-^\top \quad \text{and} \quad \begin{bmatrix} X_-\Theta & X_+\Theta \\ \Theta^\top X_+^\top & X_-\Theta \end{bmatrix} > 0.$$

Definition: We say the data $\mathcal{D}$ are informative for stabilization if there exists a $K \in \mathbb{R}^{m \times n}$ such that $A + BK$ is Schur for all $(A, B) \in \Sigma_{\mathcal{D}}$.

Proposition (vW et al., 2020): $\mathcal{D}$ are informative for stabilization if and only if $\operatorname{rank} X_- = n$ and there exists a right inverse $X_-^\dagger$ such that $X_+ X_-^\dagger$ is Schur.
In this case, $K := U_- X_-^\dagger$ is a stabilizing feedback gain for all systems in $\Sigma_{\mathcal{D}}$.

Remark: If the data are informative, a suitable $X_-^\dagger$ may be computed as follows:

- Find $\Theta \in \mathbb{R}^{T \times n}$ solving the linear matrix inequality (LMI):

$$X_- \Theta = \Theta^\top X_-^\top \quad \text{and} \quad \begin{bmatrix} X_- \Theta & X_+ \Theta \\ \Theta^\top X_+^\top & X_- \Theta \end{bmatrix} > 0.$$

- Set $X_-^\dagger := \Theta (X_- \Theta)^{-1}$.

Definition: We say the data $\mathcal{D}$ are informative for stabilization if there exists a $K \in \mathbb{R}^{m\times n}$ such that $A+BK$ is Schur for all $(A,B) \in \Sigma_{\mathcal{D}}$.

Proposition (vW et al., 2020): $\mathcal{D}$ are informative for stabilization if and only if $\operatorname{rank} X_- = n$ and there exists a right inverse $X_-^\dagger$ such that $X_+X_-^\dagger$ is Schur.
In this case, $K := U_-X_-^\dagger$ is a stabilizing feedback gain for all systems in $\Sigma_{\mathcal{D}}$.

Remark: If the data are informative, a suitable $X_-^\dagger$ may be computed as follows:

- Find $\Theta \in \mathbb{R}^{T\times n}$ solving the linear matrix inequality (LMI):

$$X_-\Theta = \Theta^\top X_-^\top \quad \text{and} \quad \begin{bmatrix} X_-\Theta & X_+\Theta \\ \Theta^\top X_+^\top & X_-\Theta \end{bmatrix} > 0.$$

- Set $X_-^\dagger := \Theta(X_-\Theta)^{-1}$.
- Finally, $K = U_-X_-^\dagger = U_-\Theta(X_-\Theta)^{-1}$.

Definition: We say the data $\mathcal{D}$ are informative for stabilization if there exists a $K \in \mathbb{R}^{m\times n}$ such that $A+BK$ is Schur for all $(A,B) \in \Sigma_{\mathcal{D}}$.

Proposition (vW et al., 2020): $\mathcal{D}$ are informative for stabilization if and only if $\operatorname{rank} X_- = n$ and there exists a right inverse $X_-^\dagger$ such that $X_+X_-^\dagger$ is Schur. In this case, $K := U_-X_-^\dagger$ is a stabilizing feedback gain for all systems in $\Sigma_{\mathcal{D}}$.

Remark: If the data are informative, a suitable $X_-^\dagger$ may be computed as follows:

- Find $\Theta \in \mathbb{R}^{T\times n}$ solving the linear matrix inequality (LMI):

$$X_-\Theta = \Theta^\top X_-^\top \quad \text{and} \quad \begin{bmatrix} X_-\Theta & X_+\Theta \\ \Theta^\top X_+^\top & X_-\Theta \end{bmatrix} > 0.$$

- Set $X_-^\dagger := \Theta(X_-\Theta)^{-1}$.
- Finally, $K = U_-X_-^\dagger = U_-\Theta(X_-\Theta)^{-1}$.

Remark: There are alternative LMIs[1] where size of variables is independent of T.

[1] vW and Camlibel, "A Matrix Finsler's Lemma with Applications to Data-Driven Control", in IEEE CDC, 2021.

Let $\Sigma_{\mathrm{pk}} \subseteq \mathcal{M}$ be a set capturing our prior knowledge of the true system, i.e.,

$$(A_{\mathrm{true}}, B_{\mathrm{true}}) \in \Sigma_{\mathrm{pk}}.$$

Let $\Sigma_{\text{pk}} \subseteq \mathcal{M}$ be a set capturing our prior knowledge of the true system, i.e.,

$$(A_{\text{true}}, B_{\text{true}}) \in \Sigma_{\text{pk}}.$$

Data-driven stabilization problem (with Σ_{pk}): Given $\mathcal{D}$ and Σ_{pk}, find $K \in \mathbb{R}^{m \times n}$ such that $A + BK$ is Schur for all $(A, B) \in \Sigma_{\mathcal{D}} \cap \Sigma_{\text{pk}}$.

Let $\Sigma_{\text{pk}} \subseteq \mathcal{M}$ be a set capturing our prior knowledge of the true system, i.e.,

$$(A_{\text{true}}, B_{\text{true}}) \in \Sigma_{\text{pk}}.$$

Data-driven stabilization problem (with Σ_{pk}): Given $\mathcal{D}$ and Σ_{pk}, find $K \in \mathbb{R}^{m \times n}$ such that $A + BK$ is Schur for all $(A, B) \in \Sigma_{\mathcal{D}} \cap \Sigma_{\text{pk}}$.

Definition: The data $\mathcal{D}$ are called Σ_{pk}-informative for stabilization if there exists $K \in \mathbb{R}^{m \times n}$ such that $A + BK$ is Schur for all $(A, B) \in \Sigma_{\mathcal{D}} \cap \Sigma_{\text{pk}}$.

Let $\Sigma_{\text{pk}} \subseteq \mathcal{M}$ be a set capturing our prior knowledge of the true system, i.e.,

$$(A_{\text{true}}, B_{\text{true}}) \in \Sigma_{\text{pk}}.$$

Data-driven stabilization problem (with Σ_{pk}): Given $\mathcal{D}$ and Σ_{pk}, find $K \in \mathbb{R}^{m\times n}$ such that $A + BK$ is Schur for all $(A, B) \in \Sigma_{\mathcal{D}} \cap \Sigma_{\text{pk}}$.

Definition: The data $\mathcal{D}$ are called Σ_{pk}-informative for stabilization if there exists $K \in \mathbb{R}^{m\times n}$ such that $A + BK$ is Schur for all $(A, B) \in \Sigma_{\mathcal{D}} \cap \Sigma_{\text{pk}}$.

This talk: Σ_{pk}-informativity for $\Sigma_{\text{pk}} = \Sigma_{\text{cont}}$ and $\Sigma_{\text{pk}} = \Sigma_{\text{stab}}$, where

$$\Sigma_{\text{cont}} := \{(A, B) \in \mathcal{M} \mid (A, B) \text{ is controllable}\}$$
$$\Sigma_{\text{stab}} := \{(A, B) \in \mathcal{M} \mid (A, B) \text{ is stabilizable}\}.$$

Let $\Sigma_{\text{pk}} \subseteq \mathcal{M}$ be a set capturing our prior knowledge of the true system, i.e.,

$$(A_{\text{true}}, B_{\text{true}}) \in \Sigma_{\text{pk}}.$$

Data-driven stabilization problem (with Σ_{pk}): Given $\mathcal{D}$ and Σ_{pk}, find $K \in \mathbb{R}^{m \times n}$ such that $A + BK$ is Schur for all $(A, B) \in \Sigma_{\mathcal{D}} \cap \Sigma_{\text{pk}}$.

Definition: The data $\mathcal{D}$ are called Σ_{pk}-informative for stabilization if there exists $K \in \mathbb{R}^{m \times n}$ such that $A + BK$ is Schur for all $(A, B) \in \Sigma_{\mathcal{D}} \cap \Sigma_{\text{pk}}$.

This talk: Σ_{pk}-informativity for $\Sigma_{\text{pk}} = \Sigma_{\text{cont}}$ and $\Sigma_{\text{pk}} = \Sigma_{\text{stab}}$, where

$$\Sigma_{\text{cont}} := \{(A, B) \in \mathcal{M} \mid (A, B) \text{ is controllable}\}$$
$$\Sigma_{\text{stab}} := \{(A, B) \in \mathcal{M} \mid (A, B) \text{ is stabilizable}\}.$$

Recall: The pair $(A, B) \in \mathcal{M}$ is called

- controllable if $\mathcal{R}(A, B) := \operatorname{im} \begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix}$ equals $\mathbb{R}^n$.
- stabilizable if there exists $F \in \mathbb{R}^{m \times n}$ such that $A + BF$ is Schur.

Let $\Sigma_{\text{pk}} \subseteq \mathcal{M}$ be a set capturing our prior knowledge of the true system, i.e.,

$$(A_{\text{true}}, B_{\text{true}}) \in \Sigma_{\text{pk}}.$$

Data-driven stabilization problem (with Σ_{pk}): Given $\mathcal{D}$ and Σ_{pk}, find $K \in \mathbb{R}^{m\times n}$ such that $A + BK$ is Schur for all $(A, B) \in \Sigma_{\mathcal{D}} \cap \Sigma_{\text{pk}}$.

Definition: The data $\mathcal{D}$ are called Σ_{pk}-informative for stabilization if there exists $K \in \mathbb{R}^{m\times n}$ such that $A + BK$ is Schur for all $(A, B) \in \Sigma_{\mathcal{D}} \cap \Sigma_{\text{pk}}$.

This talk: Σ_{pk}-informativity for $\Sigma_{\text{pk}} = \Sigma_{\text{cont}}$ and $\Sigma_{\text{pk}} = \Sigma_{\text{stab}}$, where

$$\Sigma_{\text{cont}} := \{(A, B) \in \mathcal{M} \mid (A, B) \text{ is controllable}\}$$
$$\Sigma_{\text{stab}} := \{(A, B) \in \mathcal{M} \mid (A, B) \text{ is stabilizable}\}.$$

Recall: The pair $(A, B) \in \mathcal{M}$ is called

- controllable if $\mathcal{R}(A, B) := \operatorname{im} \begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix}$ equals $\mathbb{R}^n$.
- stabilizable if there exists $F \in \mathbb{R}^{m\times n}$ such that $A + BF$ is Schur.

Note: This is challenging because $\Sigma_{\mathcal{D}} \cap \Sigma_{\text{pk}}$ is not convex!

Example: Consider the data

$$U_- = \begin{bmatrix} 1 & 2 & -1 \end{bmatrix}, \; X_- = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 0 \end{bmatrix}, \quad \text{and} \quad X_+ = \begin{bmatrix} 2 & 4 & 3 \\ 0 & 0 & 0 \end{bmatrix}.$$

Example: Consider the data

$$U_- = \begin{bmatrix} 1 & 2 & -1 \end{bmatrix}, \; X_- = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 0 \end{bmatrix}, \; \text{and} \; X_+ = \begin{bmatrix} 2 & 4 & 3 \\ 0 & 0 & 0 \end{bmatrix}.$$

We have

$$\Sigma_{\mathcal{D}} = \left\{ \left(\begin{bmatrix} 1 & \alpha \\ 0 & \beta \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) \mid \alpha, \beta \in \mathbb{R} \right\}.$$

Example: Consider the data

$$U_- = \begin{bmatrix} 1 & 2 & -1 \end{bmatrix}, \; X_- = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 0 \end{bmatrix}, \quad \text{and} \quad X_+ = \begin{bmatrix} 2 & 4 & 3 \\ 0 & 0 & 0 \end{bmatrix}.$$

We have

$$\Sigma_{\mathcal{D}} = \left\{ \left(\begin{bmatrix} 1 & \alpha \\ 0 & \beta \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) \mid \alpha, \beta \in \mathbb{R} \right\}.$$

Since $\operatorname{rank} X_- = 1 \neq 2$, $\mathcal{D}$ are not informative for stabilization.

Example: Consider the data

$$U_- = \begin{bmatrix} 1 & 2 & -1 \end{bmatrix}, \; X_- = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 0 \end{bmatrix}, \; \text{and} \; X_+ = \begin{bmatrix} 2 & 4 & 3 \\ 0 & 0 & 0 \end{bmatrix}.$$

We have

$$\Sigma_{\mathcal{D}} = \left\{ \left(\begin{bmatrix} 1 & \alpha \\ 0 & \beta \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) \mid \alpha, \beta \in \mathbb{R} \right\}.$$

Since $\operatorname{rank} X_- = 1 \neq 2$, $\mathcal{D}$ are not informative for stabilization.

However, we have

$$\Sigma_{\mathcal{D}} \cap \Sigma_{\text{stab}} = \left\{ \left(\begin{bmatrix} 1 & \alpha \\ 0 & \beta \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) \mid \alpha \in \mathbb{R}, \; |\beta| < 1 \right\}.$$

Example: Consider the data

$$U_- = \begin{bmatrix} 1 & 2 & -1 \end{bmatrix}, \; X_- = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 0 \end{bmatrix}, \; \text{and} \; X_+ = \begin{bmatrix} 2 & 4 & 3 \\ 0 & 0 & 0 \end{bmatrix}.$$

We have

$$\Sigma_{\mathcal{D}} = \left\{ \left(\begin{bmatrix} 1 & \alpha \\ 0 & \beta \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) \mid \alpha, \beta \in \mathbb{R} \right\}.$$

Since $\operatorname{rank} X_- = 1 \neq 2$, $\mathcal{D}$ are not informative for stabilization.

However, we have

$$\Sigma_{\mathcal{D}} \cap \Sigma_{\text{stab}} = \left\{ \left(\begin{bmatrix} 1 & \alpha \\ 0 & \beta \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) \mid \alpha \in \mathbb{R}, \; |\beta| < 1 \right\}.$$

It is evident that $K = \begin{bmatrix} -1 & 0 \end{bmatrix}$ stabilizes all systems in $\Sigma_{\mathcal{D}} \cap \Sigma_{\text{stab}}$.

Example: Consider the data

$$U_- = \begin{bmatrix} 1 & 2 & -1 \end{bmatrix}, \; X_- = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 0 \end{bmatrix}, \text{ and } X_+ = \begin{bmatrix} 2 & 4 & 3 \\ 0 & 0 & 0 \end{bmatrix}.$$

We have

$$\Sigma_{\mathcal{D}} = \left\{ \left(\begin{bmatrix} 1 & \alpha \\ 0 & \beta \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) \mid \alpha, \beta \in \mathbb{R} \right\}.$$

Since $\operatorname{rank} X_- = 1 \neq 2$, $\mathcal{D}$ are not informative for stabilization.

However, we have

$$\Sigma_{\mathcal{D}} \cap \Sigma_{\text{stab}} = \left\{ \left(\begin{bmatrix} 1 & \alpha \\ 0 & \beta \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) \mid \alpha \in \mathbb{R}, \; |\beta| < 1 \right\}.$$

It is evident that $K = \begin{bmatrix} -1 & 0 \end{bmatrix}$ stabilizes all systems in $\Sigma_{\mathcal{D}} \cap \Sigma_{\text{stab}}$.

Therefore, $\mathcal{D}$ are Σ_{stab}–informative for stabilization.

We focus on two types of prior knowledge:

$$\Sigma_{\text{pk}} = \Sigma_{\text{cont}} \quad \text{and} \quad \Sigma_{\text{pk}} = \Sigma_{\text{stab}}.$$

We focus on two types of prior knowledge:

$$\Sigma_{\text{pk}} = \Sigma_{\text{cont}} \quad \text{and} \quad \Sigma_{\text{pk}} = \Sigma_{\text{stab}}.$$

Problem: Find necessary and sufficient conditions under which the data $\mathcal{D}$ are

(i) Σ_{cont}–informative for stabilization;

(ii) Σ_{stab}–informative for stabilization.

Theorem: Suppose that $(A_{\text{true}}, B_{\text{true}}) \in \Sigma_{\text{cont}}$.

Theorem: Suppose that $(A_{\text{true}}, B_{\text{true}}) \in \Sigma_{\text{cont}}$. Then, the following statements are equivalent:

(a) $\mathcal{D}$ are Σ_{cont}–informative for stabilization.

(b) $\mathcal{D}$ are informative for stabilization.

Theorem: Suppose that $(A_{\rm true}, B_{\rm true}) \in \Sigma_{\rm cont}$. Then, the following statements are equivalent:

(a) $\mathcal{D}$ are $\Sigma_{\rm cont}$–informative for stabilization.

(b) $\mathcal{D}$ are informative for stabilization.

Moreover, if K is such that $A + BK$ is Schur for all $(A, B) \in \Sigma_{\mathcal{D}} \cap \Sigma_{\rm cont}$, then $A + BK$ is Schur for all $(A, B) \in \Sigma_{\mathcal{D}}$.

Theorem: Suppose that $(A_{\mathrm{true}}, B_{\mathrm{true}}) \in \Sigma_{\mathrm{cont}}$. Then, the following statements are equivalent:

(a) $\mathcal{D}$ are Σ_{cont}–informative for stabilization.

(b) $\mathcal{D}$ are informative for stabilization.

Moreover, if K is such that $A + BK$ is Schur for all $(A, B) \in \Sigma_{\mathcal{D}} \cap \Sigma_{\mathrm{cont}}$, then $A + BK$ is Schur for all $(A, B) \in \Sigma_{\mathcal{D}}$.

Interpretation: Prior knowledge on controllability does not help!

Theorem: Suppose that $(A_{\text{true}}, B_{\text{true}}) \in \Sigma_{\text{cont}}$. Then, the following statements are equivalent:

(a) $\mathcal{D}$ are Σ_{cont}–informative for stabilization.

(b) $\mathcal{D}$ are informative for stabilization.

Moreover, if K is such that $A + BK$ is Schur for all $(A, B) \in \Sigma_{\mathcal{D}} \cap \Sigma_{\text{cont}}$, then $A + BK$ is Schur for all $(A, B) \in \Sigma_{\mathcal{D}}$.

Interpretation: Prior knowledge on controllability does not help!

Remark: The theorem is based on the following two lemmas:

Theorem: Suppose that $(A_{\text{true}}, B_{\text{true}}) \in \Sigma_{\text{cont}}$. Then, the following statements are equivalent:

(a) $\mathcal{D}$ are Σ_{cont}–informative for stabilization.

(b) $\mathcal{D}$ are informative for stabilization.

Moreover, if K is such that $A + BK$ is Schur for all $(A, B) \in \Sigma_{\mathcal{D}} \cap \Sigma_{\text{cont}}$, then $A + BK$ is Schur for all $(A, B) \in \Sigma_{\mathcal{D}}$.

Interpretation: Prior knowledge on controllability does not help!

Remark: The theorem is based on the following two lemmas:

Lemma 1: Let $(M, N) \in \Sigma_{\text{cont}}$ and $(M_0, N_0) \in \mathcal{M}$. Then, the pair $(M + \alpha M_0, N + \alpha N_0)$ is controllable for all but at most n^2 values of $\alpha \in \mathbb{R}$.

Theorem: Suppose that $(A_{\rm true}, B_{\rm true}) \in \Sigma_{\rm cont}$. Then, the following statements are equivalent:

(a) $\mathcal{D}$ are $\Sigma_{\rm cont}$–informative for stabilization.

(b) $\mathcal{D}$ are informative for stabilization.

Moreover, if K is such that $A + BK$ is Schur for all $(A, B) \in \Sigma_{\mathcal{D}} \cap \Sigma_{\rm cont}$, then $A + BK$ is Schur for all $(A, B) \in \Sigma_{\mathcal{D}}$.

Interpretation: Prior knowledge on controllability does not help!

Remark: The theorem is based on the following two lemmas:

Lemma 1: Let $(M, N) \in \Sigma_{\rm cont}$ and $(M_0, N_0) \in \mathcal{M}$. Then, the pair $(M + \alpha M_0, N + \alpha N_0)$ is controllable for all but at most n^2 values of $\alpha \in \mathbb{R}$.

Lemma 2: Let $\varepsilon \in \mathbb{R}$, $\mathcal{F} \subset \mathbb{R}$ be a finite set, and $M, N \in \mathbb{R}^{n\times n}$ be such that $M + \delta N$ is Schur for all $\delta \in [\varepsilon, \infty)\backslash\mathcal{F}$. Then, $M + \delta N$ is Schur for all $\delta \in \mathbb{R}$.

Theorem (necessary conditions): Suppose that $(A_{\text{true}}, B_{\text{true}}) \in \Sigma_{\text{stab}}$ and $\mathcal{D}$ are Σ_{stab}–informative for stabilization.

Theorem (necessary conditions): Suppose that $(A_{\rm true}, B_{\rm true}) \in \Sigma_{\rm stab}$ and $\mathcal{D}$ are $\Sigma_{\rm stab}$–informative for stabilization. Then the following statements hold:

(a) $\operatorname{im} X_+ \subseteq \operatorname{im} X_-$.

Theorem (necessary conditions): Suppose that $(A_{\text{true}}, B_{\text{true}}) \in \Sigma_{\text{stab}}$ and $\mathcal{D}$ are Σ_{stab}–informative for stabilization. Then the following statements hold:

(a) $\operatorname{im} X_+ \subseteq \operatorname{im} X_-$.

(b) If $\operatorname{rank} X_- < n$, then $\operatorname{im} \begin{bmatrix} X_- \\ U_- \end{bmatrix} = \operatorname{im} X_- \times \mathbb{R}^m$.

Theorem (necessary conditions): Suppose that $(A_{\text{true}}, B_{\text{true}}) \in \Sigma_{\text{stab}}$ and $\mathcal{D}$ are Σ_{stab}–informative for stabilization. Then the following statements hold:

(a) $\operatorname{im} X_+ \subseteq \operatorname{im} X_-$.

(b) If $\operatorname{rank} X_- < n$, then $\operatorname{im} \begin{bmatrix} X_- \\ U_- \end{bmatrix} = \operatorname{im} X_- \times \mathbb{R}^m$. ($\Rightarrow B_{\text{true}}$ can be recovered)

Recall: If $\mathcal{D}$ are informative for stabilization (without prior knowledge), then

$$\operatorname{im} X_- = \mathbb{R}^n.$$

Theorem (necessary conditions): Suppose that $(A_{\text{true}}, B_{\text{true}}) \in \Sigma_{\text{stab}}$ and $\mathcal{D}$ are Σ_{stab}–informative for stabilization. Then the following statements hold:

(a) $\operatorname{im} X_+ \subseteq \operatorname{im} X_-$.

(b) If $\operatorname{rank} X_- < n$, then $\operatorname{im} \begin{bmatrix} X_- \\ U_- \end{bmatrix} = \operatorname{im} X_- \times \mathbb{R}^m$. ($\Rightarrow$ B_{true} can be recovered)

Recall: If $\mathcal{D}$ are informative for stabilization (without prior knowledge), then

$$\operatorname{im} X_- = \mathbb{R}^n.$$

Theorem (necessary and sufficient conditions): Suppose that $(A_{\text{true}}, B_{\text{true}}) \in \Sigma_{\text{stab}}$. Then:

Theorem (necessary conditions): Suppose that $(A_{\text{true}}, B_{\text{true}}) \in \Sigma_{\text{stab}}$ and $\mathcal{D}$ are Σ_{stab}–informative for stabilization. Then the following statements hold:

(a) $\operatorname{im} X_+ \subseteq \operatorname{im} X_-$.

(b) If $\operatorname{rank} X_- < n$, then $\operatorname{im} \begin{bmatrix} X_- \\ U_- \end{bmatrix} = \operatorname{im} X_- \times \mathbb{R}^m$. ($\Rightarrow$ B_{true} can be recovered)

Recall: If $\mathcal{D}$ are informative for stabilization (without prior knowledge), then

$$\operatorname{im} X_- = \mathbb{R}^n.$$

Theorem (necessary and sufficient conditions): Suppose that $(A_{\text{true}}, B_{\text{true}}) \in \Sigma_{\text{stab}}$. Then:

- If $\operatorname{rank} X_- = n$, the following are equivalent:
  - (i) $\mathcal{D}$ are Σ_{stab}–informative for stabilization.
  - (ii) $\mathcal{D}$ are informative for stabilization.

Theorem (necessary conditions): Suppose that $(A_{\text{true}}, B_{\text{true}}) \in \Sigma_{\text{stab}}$ and $\mathcal{D}$ are Σ_{stab}–informative for stabilization. Then the following statements hold:

(a) $\operatorname{im} X_+ \subseteq \operatorname{im} X_-$.

(b) If $\operatorname{rank} X_- < n$, then $\operatorname{im} \begin{bmatrix} X_- \\ U_- \end{bmatrix} = \operatorname{im} X_- \times \mathbb{R}^m$. ($\Rightarrow$ B_{true} can be recovered)

Recall: If $\mathcal{D}$ are informative for stabilization (without prior knowledge), then

$$\operatorname{im} X_- = \mathbb{R}^n.$$

Theorem (necessary and sufficient conditions): Suppose that $(A_{\text{true}}, B_{\text{true}}) \in \Sigma_{\text{stab}}$. Then:

- If $\operatorname{rank} X_- = n$, the following are equivalent:
  - (i) $\mathcal{D}$ are Σ_{stab}–informative for stabilization.
  - (ii) $\mathcal{D}$ are informative for stabilization.
- If $\operatorname{rank} X_- < n$ then $\mathcal{D}$ are Σ_{stab}–informative for stabilization if and only if
  - (1) $\operatorname{im} X_+ \subseteq \operatorname{im} X_-$, and
  - (2) $\operatorname{im} \begin{bmatrix} X_- \\ U_- \end{bmatrix} = \operatorname{im} X_- \times \mathbb{R}^m$.

Example (revisited): Consider the data

$$U_- = \begin{bmatrix} 1 & 2 & -1 \end{bmatrix}, \; X_- = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 0 \end{bmatrix}, \quad \text{and} \quad X_+ = \begin{bmatrix} 2 & 4 & 3 \\ 0 & 0 & 0 \end{bmatrix}.$$

Example (revisited): Consider the data

$$U_- = \begin{bmatrix} 1 & 2 & -1 \end{bmatrix}, \; X_- = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 0 \end{bmatrix}, \text{ and } X_+ = \begin{bmatrix} 2 & 4 & 3 \\ 0 & 0 & 0 \end{bmatrix}.$$

$$\operatorname{im} X_- = \operatorname{im} X_+ = \operatorname{im} \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Example (revisited): Consider the data

$$U_- = \begin{bmatrix} 1 & 2 & -1 \end{bmatrix}, \quad X_- = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 0 \end{bmatrix}, \quad \text{and} \quad X_+ = \begin{bmatrix} 2 & 4 & 3 \\ 0 & 0 & 0 \end{bmatrix}.$$

$\operatorname{im} X_- = \operatorname{im} X_+ = \operatorname{im} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Also, $\operatorname{im} \begin{bmatrix} X_- \\ U_- \end{bmatrix} = \operatorname{im} \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} = \operatorname{im} X_- \times \mathbb{R}$.

Example (revisited): Consider the data

$$U_- = \begin{bmatrix} 1 & 2 & -1 \end{bmatrix}, \; X_- = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 0 \end{bmatrix}, \text{ and } X_+ = \begin{bmatrix} 2 & 4 & 3 \\ 0 & 0 & 0 \end{bmatrix}.$$

$\operatorname{im} X_- = \operatorname{im} X_+ = \operatorname{im} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Also, $\operatorname{im} \begin{bmatrix} X_- \\ U_- \end{bmatrix} = \operatorname{im} \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} = \operatorname{im} X_- \times \mathbb{R}$.

So the data $\mathcal{D}$ are Σ_{stab}-informative for stabilization!

Example (revisited): Consider the data

$$U_- = \begin{bmatrix} 1 & 2 & -1 \end{bmatrix}, \quad X_- = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 0 \end{bmatrix}, \quad \text{and} \quad X_+ = \begin{bmatrix} 2 & 4 & 3 \\ 0 & 0 & 0 \end{bmatrix}.$$

$\operatorname{im} X_- = \operatorname{im} X_+ = \operatorname{im} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Also, $\operatorname{im} \begin{bmatrix} X_- \\ U_- \end{bmatrix} = \operatorname{im} \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} = \operatorname{im} X_- \times \mathbb{R}$.

So the data $\mathcal{D}$ are Σ_{stab}-informative for stabilization!

Discussion: The conditions for the case $\operatorname{rank} X_- < n$ are very important in situations where $(A_{\text{true}}, B_{\text{true}}) \notin \Sigma_{\text{cont}}$.

Example (revisited): Consider the data

$$U_- = \begin{bmatrix} 1 & 2 & -1 \end{bmatrix}, \; X_- = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 0 \end{bmatrix}, \; \text{and} \; X_+ = \begin{bmatrix} 2 & 4 & 3 \\ 0 & 0 & 0 \end{bmatrix}.$$

$\operatorname{im} X_- = \operatorname{im} X_+ = \operatorname{im} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Also, $\operatorname{im} \begin{bmatrix} X_- \\ U_- \end{bmatrix} = \operatorname{im} \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} = \operatorname{im} X_- \times \mathbb{R}$.

So the data $\mathcal{D}$ are Σ_{stab}-informative for stabilization!

Discussion: The conditions for the case $\operatorname{rank} X_- < n$ are very important in situations where $(A_{\text{true}}, B_{\text{true}}) \notin \Sigma_{\text{cont}}$.
Indeed, in this case, when $x(0) = 0$,

$$\operatorname{rank} X_- < n$$

for any sequence of inputs $u(0), u(1), \ldots, u(T-1)$.

Example (revisited): Consider the data

$$U_- = \begin{bmatrix} 1 & 2 & -1 \end{bmatrix}, \quad X_- = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 0 \end{bmatrix}, \quad \text{and} \quad X_+ = \begin{bmatrix} 2 & 4 & 3 \\ 0 & 0 & 0 \end{bmatrix}.$$

$\operatorname{im} X_- = \operatorname{im} X_+ = \operatorname{im} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Also, $\operatorname{im} \begin{bmatrix} X_- \\ U_- \end{bmatrix} = \operatorname{im} \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} = \operatorname{im} X_- \times \mathbb{R}$.

So the data $\mathcal{D}$ are Σ_{stab}-informative for stabilization!

Discussion: The conditions for the case $\operatorname{rank} X_- < n$ are very important in situations where $(A_{\text{true}}, B_{\text{true}}) \notin \Sigma_{\text{cont}}$.
Indeed, in this case, when $x(0) = 0$,

$$\operatorname{rank} X_- < n$$

for any sequence of inputs $u(0), u(1), \dots, u(T-1)$.

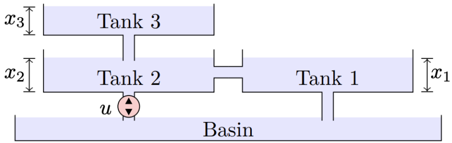
Remark: In case $\operatorname{rank} X_- < n$, there are also LMI methods[2] for constructing K!

[2] Shakouri, vW, Baltussen, and Heemels, "Data-Driven Stabilization Using Prior Knowledge on Stabilizability and Controllability", submitted, 2025.

We consider a (discretized) model of a three-tank system:

$$A_{\text{true}} = \begin{bmatrix} 0.9429 & 0.0473 & 0.0012 \\ 0.0473 & 0.9524 & 0.0476 \\ 0 & 0 & 0.9512 \end{bmatrix},$$

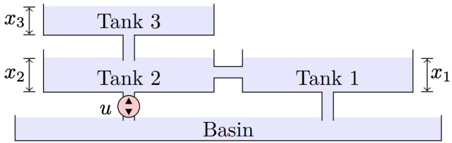
$$B_{\text{true}} = \begin{bmatrix} 0.0024 \\ 0.0976 \\ 0 \end{bmatrix}.$$



We consider a (discretized) model of a three-tank system:

$$A_{\text{true}} = \begin{bmatrix} 0.9429 & 0.0473 & 0.0012 \\ 0.0473 & 0.9524 & 0.0476 \\ 0 & 0 & 0.9512 \end{bmatrix},$$

$$B_{\text{true}} = \begin{bmatrix} 0.0024 \\ 0.0976 \\ 0 \end{bmatrix}.$$

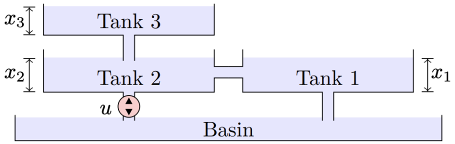


1. Monte Carlo simulations with 1000 scenarios;

We consider a (discretized) model of a three-tank system:

$$A_{\text{true}} = \begin{bmatrix} 0.9429 & 0.0473 & 0.0012 \\ 0.0473 & 0.9524 & 0.0476 \\ 0 & 0 & 0.9512 \end{bmatrix},$$

$$B_{\text{true}} = \begin{bmatrix} 0.0024 \\ 0.0976 \\ 0 \end{bmatrix}.$$

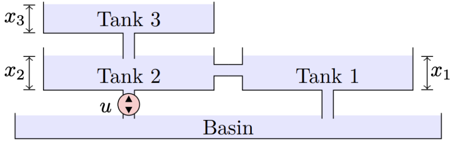


---

1. Monte Carlo simulations with 1000 scenarios;
2. For each scenario,
   - we simulate the system from $t = 0$ to $t = 100$;
   - the input and initial condition $\sim$ Poisson distribution with parameter $\lambda = 1$.
3. We use the first T samples for each round of analysis; $T = 3, 4, 5, 10, 100$.

$$A_{\text{true}} = \begin{bmatrix} 0.9429 & 0.0473 & 0.0012 \\ 0.0473 & 0.9524 & 0.0476 \\ 0 & 0 & 0.9512 \end{bmatrix},$$

$$B_{\text{true}} = \begin{bmatrix} 0.0024 \\ 0.0976 \\ 0 \end{bmatrix}.$$



$$A_{\text{true}} = \begin{bmatrix} 0.9429 & 0.0473 & 0.0012 \\ 0.0473 & 0.9524 & 0.0476 \\ 0 & 0 & 0.9512 \end{bmatrix},$$

$$B_{\text{true}} = \begin{bmatrix} 0.0024 \\ 0.0976 \\ 0 \end{bmatrix}.$$

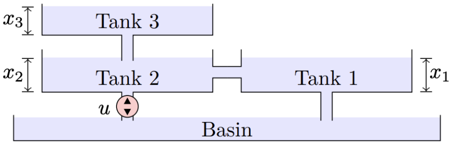


Table: Informativity of randomly generated data.

T	Informative for system identification	Σ_{pk}–informative for stabilization	
		$\Sigma_{\text{pk}} = \mathcal{M}$	$\Sigma_{\text{pk}} = \Sigma_{\text{stab}}$
3	0%	8.1%	42%
4	62.4%	63.2%	99.4%
5	62.8%	63.2%	99.8%
10	63.2%	63.2%	100%
100	63.2%	63.2%	100%

Question: Given $x(0) \in \mathbb{R}^n$, how to choose $u(0), u(1), \ldots, u(T-1) \in \mathbb{R}^m$ such that the resulting data $\mathcal{D} = (U_-, X)$ are Σ_{pk}-informative for stabilization?

Question: Given $x(0) \in \mathbb{R}^n$, how to choose $u(0), u(1), \ldots, u(T-1) \in \mathbb{R}^m$ such that the resulting data $\mathcal{D} = (U_-, X)$ are Σ_{pk}-informative for stabilization?

Let $X_{[0,t-1]} := \begin{bmatrix} x(0) & \cdots & x(t-1) \end{bmatrix}$ and $U_{[0,t-1]} := \begin{bmatrix} u(0) & \cdots & u(t-1) \end{bmatrix}$.

Question: Given $x(0) \in \mathbb{R}^n$, how to choose $u(0), u(1), \ldots, u(T-1) \in \mathbb{R}^m$ such that the resulting data $\mathcal{D} = (U_-, X)$ are Σ_{pk}-informative for stabilization?

Let $X_{[0,t-1]} := \begin{bmatrix} x(0) & \cdots & x(t-1) \end{bmatrix}$ and $U_{[0,t-1]} := \begin{bmatrix} u(0) & \cdots & u(t-1) \end{bmatrix}$.

Algorithm 1 Online experiment design

```
Given: x(0) ∈ ℝ^n
set t ← 0 and select u(0) ≠ 0
while true do
    apply u(t) to (A_true, B_true) and measure x(t+1)
    set t ← t+1
    if x(t) ∉ im X_[0,t-1] then
        select u(t) arbitrarily
    else
        if im [X_[0,t-1]; U_[0,t-1]] = im X_[0,t-1] × ℝ^m then
            break
        else
            select ξ ∈ ℝ^n and η ∈ ℝ^m \ {0} such that
                ξ^⊤ X_[0,t-1] + η^⊤ U_[0,t-1] = 0
            select u(t) such that ξ^⊤ x(t) + η^⊤ u(t) ≠ 0
        end if
    end if
end while
Return: T = t and D = (U_[0,T-1], X_[0,T])
```

Line 8 condition: $\operatorname{im} \begin{bmatrix} X_{[0,t-1]} \\ U_{[0,t-1]} \end{bmatrix} = \operatorname{im} X_{[0,t-1]} \times \mathbb{R}^m$

Line 11 condition: $\xi^\top X_{[0,t-1]} + \eta^\top U_{[0,t-1]} = 0$

Return: $T = t$ and $\mathcal{D} = (U_{[0,T-1]}, X_{[0,T]})$

Experiment design for stabilization

Algorithm 1 Online experiment design

Given: $x(0) \in \mathbb{R}^n$

set $t \leftarrow 0$ and select $u(0) \neq 0$
while true **do**
  apply $u(t)$ to $(A_{\text{true}}, B_{\text{true}})$ and measure $x(t+1)$
  set $t \leftarrow t+1$
  if $x(t) \notin \operatorname{im} X_{[0,t-1]}$ **then**
    select $u(t)$ arbitrarily
  else
    if $\operatorname{im} \begin{bmatrix} X_{[0,t-1]} \\ U_{[0,t-1]} \end{bmatrix} = \operatorname{im} X_{[0,t-1]} \times \mathbb{R}^m$ **then**
      break
    else
      select $\xi \in \mathbb{R}^n$ and $\eta \in \mathbb{R}^m \backslash \{0\}$ such that

$$\xi^\top X_{[0,t-1]} + \eta^\top U_{[0,t-1]} = 0$$

      select $u(t)$ such that $\xi^\top x(t) + \eta^\top u(t) \neq 0$
    end if
  end if
end while

Return: $T = t$ and $\mathcal{D} = (U_{[0,T-1]}, X_{[0,T]})$

Algorithm 1 Online experiment design

Given: $x(0) \in \mathbb{R}^n$

set $t \leftarrow 0$ and select $u(0) \neq 0$
while true **do**
  apply $u(t)$ to $(A_{\text{true}}, B_{\text{true}})$ and measure $x(t+1)$
  set $t \leftarrow t+1$
  if $x(t) \notin \operatorname{im} X_{[0,t-1]}$ **then**
    select $u(t)$ arbitrarily
  else
    if $\operatorname{im} \begin{bmatrix} X_{[0,t-1]} \\ U_{[0,t-1]} \end{bmatrix} = \operatorname{im} X_{[0,t-1]} \times \mathbb{R}^m$ **then**
      break
    else
      select $\xi \in \mathbb{R}^n$ and $\eta \in \mathbb{R}^m \setminus \{0\}$ such that

$$\xi^\top X_{[0,t-1]} + \eta^\top U_{[0,t-1]} = 0$$

      select $u(t)$ such that $\xi^\top x(t) + \eta^\top u(t) \neq 0$
    end if
  end if
end while

Return: $T = t$ and $\mathcal{D} = (U_{[0,T-1]}, X_{[0,T]})$

Theorem: Let $\mathcal{D}$ be generated by Algorithm 1. Then:

1. $T = \dim \mathcal{R}(A_{\text{true}}, \begin{bmatrix} B_{\text{true}} & x(0) \end{bmatrix}) + m \leqslant n + m.$

Experiment design for stabilization

Algorithm 1 Online experiment design

Given: $x(0) \in \mathbb{R}^n$

set $t \leftarrow 0$ and select $u(0) \neq 0$
while true **do**
  apply $u(t)$ to $(A_{\text{true}}, B_{\text{true}})$ and measure $x(t+1)$
  set $t \leftarrow t+1$
  if $x(t) \notin \operatorname{im} X_{[0,t-1]}$ **then**
    select $u(t)$ arbitrarily
  else
    if $\operatorname{im} \begin{bmatrix} X_{[0,t-1]} \\ U_{[0,t-1]} \end{bmatrix} = \operatorname{im} X_{[0,t-1]} \times \mathbb{R}^m$ **then**
      break
    else
      select $\xi \in \mathbb{R}^n$ and $\eta \in \mathbb{R}^m \backslash \{0\}$ such that
      $$\xi^\top X_{[0,t-1]} + \eta^\top U_{[0,t-1]} = 0$$
      select $u(t)$ such that $\xi^\top x(t) + \eta^\top u(t) \neq 0$
    end if
  end if
end while

Return: $T = t$ and $\mathcal{D} = (U_{[0,T-1]}, X_{[0,T]})$

Theorem: Let $\mathcal{D}$ be generated by Algorithm 1. Then:

1. $T = \dim \mathcal{R}(A_{\text{true}}, \begin{bmatrix} B_{\text{true}} & x(0) \end{bmatrix}) + m \leqslant n + m$.
2. If $(A_{\text{true}}, B_{\text{true}}) \in \Sigma_{\text{stab}}$, then $\mathcal{D}$ is Σ_{stab}–informative for stabilization.

Algorithm 1 Online experiment design

Given: $x(0) \in \mathbb{R}^n$

set $t \leftarrow 0$ and select $u(0) \neq 0$
while true **do**
  apply $u(t)$ to $(A_{\text{true}}, B_{\text{true}})$ and measure $x(t+1)$
  set $t \leftarrow t+1$
  if $x(t) \notin \operatorname{im} X_{[0,t-1]}$ **then**
    select $u(t)$ arbitrarily
  else
    if $\operatorname{im} \begin{bmatrix} X_{[0,t-1]} \\ U_{[0,t-1]} \end{bmatrix} = \operatorname{im} X_{[0,t-1]} \times \mathbb{R}^m$ **then**
      break
    else
      select $\xi \in \mathbb{R}^n$ and $\eta \in \mathbb{R}^m \backslash \{0\}$ such that
      $$\xi^\top X_{[0,t-1]} + \eta^\top U_{[0,t-1]} = 0$$
      select $u(t)$ such that $\xi^\top x(t) + \eta^\top u(t) \neq 0$
    end if
  end if
end while

Return: $T = t$ and $\mathcal{D} = (U_{[0,T-1]}, X_{[0,T]})$

Theorem: Let $\mathcal{D}$ be generated by Algorithm 1. Then:

1. $T = \dim \mathcal{R}(A_{\text{true}}, \begin{bmatrix} B_{\text{true}} & x(0) \end{bmatrix}) + m \leqslant n + m$.
2. If $(A_{\text{true}}, B_{\text{true}}) \in \Sigma_{\text{stab}}$, then $\mathcal{D}$ is Σ_{stab}–informative for stabilization.

Remark: For certain A_{true}, B_{true} and $x(0)$, Algorithm 1 even returns the **shortest experiments** for stabilization!

Take-away messages: For data-driven stabilization:

1. controllability as prior knowledge is not helpful

Take-away messages: For data-driven stabilization:

1. controllability as prior knowledge is not helpful
2. prior knowledge on stabilizability leads to weaker conditions

Take-away messages: For data-driven stabilization:

1. controllability as prior knowledge is not helpful
2. prior knowledge on stabilizability leads to weaker conditions
3. an online experiment design algorithm can generate informative data

Take-away messages: For data-driven stabilization:

1. controllability as prior knowledge is not helpful
2. prior knowledge on stabilizability leads to weaker conditions
3. an online experiment design algorithm can generate informative data

Future work:

- other types of prior knowledge

Take-away messages: For data-driven stabilization:

1. controllability as prior knowledge is not helpful
2. prior knowledge on stabilizability leads to weaker conditions
3. an online experiment design algorithm can generate informative data

Future work:

- other types of prior knowledge
- noisy data

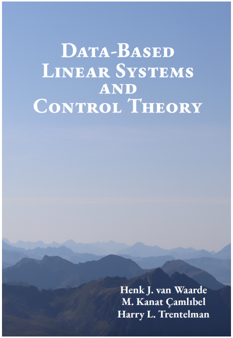
Take-away messages: For data-driven stabilization:

1. controllability as prior knowledge is not helpful
2. prior knowledge on stabilizability leads to weaker conditions
3. an online experiment design algorithm can generate informative data

Future work:

- other types of prior knowledge
- noisy data

Further reading:



Thanks to my collaborators: **Amir**, **Tren**, **Maurice** and **Kanat**!

Data-Driven Stabilization Using Prior Knowledge on Stabilizability and Controllability

Amir Shakouri, Henk J. van Waarde, Tren M.J.T. Baltussen, W.P.M.H. (Maurice) Heemels

***Abstract*— In this work, we study data-driven stabilization of linear time-invariant systems using prior knowledge of system-theoretic properties, specifically stabilizability and controllability. To formalize this, we extend the concept of data informativity by requiring the existence of a controller that stabilizes all systems**

data-driven predictive control using frequency-domain data has also been studied in [11].

The majority of papers on data-driven control work in the setting where the parameters of the system are completely unknown, which

---

Experiment design using prior knowledge on controllability and stabilizability

Amir Shakouri * **Henk J. van Waarde** *,⋆ **M. Kanat Camlibel** *

* *Bernoulli Institute for Mathematics, Computer Science and Artificial Intelligence, University of Groningen, The Netherlands (e-mail:* a.shakouri@rug.nl, h.j.van.waarde@rug.nl, m.k.camlibel@rug.nl*).*

Abstract: In this paper, we consider the problem of designing input signals for an unknown linear time-invariant system in such a way that the resulting input-state data is suitable for identification or stabilization. We will take into account prior knowledge on system-theoretic

Thank you!